3.1: Mode, Median, and Mean

_____ – the value or property that occurs most frequently in the data.

1. there can be more than 1 mode or no mode at all
2. only should be used if you are interested in the most common value

_____ – central value of an ordered distribution.

1. uses the position rather than the specific value of each data entry.

_____ – average that uses the exact value of each entry.

2. most important, but can be affected by _____.

Mode – value or property that occurs _____ in the data, but it is not the most stable way of looking at the data.

Example: 4, 6, 6, 7, 8, 9

Example: 2, 3, 4, 5, 6, 7

To Obtain the Median:

1) Put data into order from least to greatest.
2) Choose the middle value.

**If there is not one single middle value use the formula:

*** If there are an odd number of values in your distribution, the central value ____the median.

Ex. 12, 13, 16, 17, 19, 22, 35, 44, 59

*** If there is an even number of values in your distribution, obtain the median by taking the _____ of the two central values.

Ex. 27, 35, 44, 56, 67, 78, 89

To Obtain the Mean:

1) Find the _____ of all data values.
2) _____ by the total number of data entries.

There is a common notation that indicates a **sum** is the Greek letter _____
If you were to see _____, you would read that has the sum of all given x values. _____ is the total number of entries.

Mean – average that uses the _____ of each entry.
- can be affected by _____.

Ex. 113, 116, 125, 135, 110, 109, 100

Try this example:

Each month during a period of 2 years, Mel traveled the following amount of miles to work:

24	45	23	66	42	45	23
36	33	31	30	31	43	22
23	34	32	45	41	40	42
26	53	11				

a) What is the value of *n*? _____

b) What is the value of Σx? _____

c) Compute the mean (x bar). _____

For each of the following, calculate the mode, median, and mean. Round to the nearest tenth if necessary!

b. 44, 78, 91, 111, 86, 52, 57, 67, 108, 138, 11, 67, 92, 88, 75, 82, 79, 106, 111, 111, 134, 222, 45, 74, 111, 67, 92, & 45.

c. 567, 671, 670, 733, 563, 563, 672, 777, 782, 645, 375, 226, 973, 567, 711, 896, 678, 722, 917, 888, 777, 666, 335, 762, & 937.

d. 256, 102, 673, 834, 883, 991, 202, 907, 563, 444, 167, 783, 927, 863, 723, 829, 283, 923, 829, 839, 903, 920, 526, 673, 738, 672, 721, 452, & 233.

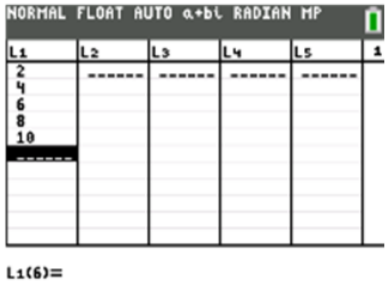
e. 1024, 1089, 8923, 6745, 6723, 7745, 8930, 4567, 8934, 8374, 8738, 8397, 7378, 9374, 7837, 8922, 2222, 7838, 7836, 7384, 7384, 7384, 7238, 2893, 2678, 2893, 8298, 7872, 2737, 1102, & 2233.

e. 4444, 2891, 1727, 2828, 4949, 2780, 2834, 4544, 4564. 1371, 7383, 2767, 2893, 8928, 2839, 7278, 2829, 9239, 8289, 2892, 8928, 1561 & 3330.

Calculator Instructions

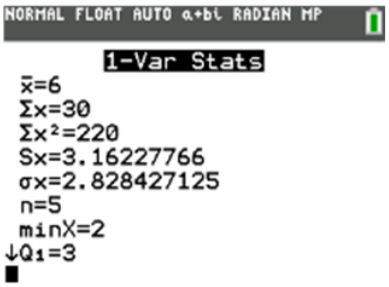
1) STAT EDIT: Choice 1 on list of options

2) Enter the data in L1



3) STAT > CALC: Choice 1 (1-Var Stats)

4) This will give the Mean (x bar), the Sum of the data Values (Σx), Number of Values (n), Median (med). You need to scroll down to see the median. However, you will need to find the Mode on your own.



Resistant Measures – one that is _____ by extremely high or low data values.

1) The mean is _____ a resistant measure of center because we can make the mean as large as we want by increasing the size of only one data value.

2) The median is _____ resistant; it is not sensitive to the specific size of a data value.

Trimmed Mean – a measure of center that is _____ than the mean but still sensitive to specific data values.

1) Eliminates the influence of unusually small or large data values.

To Compute a 5% Trimmed Mean:

1) _____ the data from smallest to largest.
2) _____ the bottom 5% of the data and the top 5% of the data.
3) _____ the mean of the remaining 90% of the data.

*** We will also be looking at 10% trimmed means

Barron's Profiles of American Colleges, 19th Edition, lists average class size for introductory lecture courses at each of the profiled institutions. A sample of 20 colleges and universities in California showed class sizes for introductory lecture courses to be

(14)	20	20	20	20	23	25	30	30	30
35	35	35	40	40	42	50	50	80	(80)

(a) Compute the mean for the entire sample. ⇨ Add all the values and divide by 20:

$$\bar{x} = \frac{\Sigma x}{n} = \frac{719}{20} \approx 36.0$$

(b) Compute a 5% trimmed mean for the sample. ⇨ The data are already ordered. Since 5% of 20 is 1, we eliminate one data value from the bottom of the list and one from the top. These values are circled in the data set. Then take the mean of the remaining 18 entries.

$$5\% \text{ trimmed mean} = \frac{\Sigma x}{n} = \frac{625}{18} \approx 34.7$$

Examples:

For each of the following compute a 5% trimmed mean and a 10% trimmed mean.

1) 98 90 98 90 90 98 97 95 87 90 65 80
79 98 89 07 80 68 88 98

2) 62 74 36 89 61 29 86 58 87 46 59 87
36 57 89 46 37 58 63 49

3) 67 65 98 61 23 56 43 69 85 23 85 63
82 65 98 32 65 89 43 65

4) 128 927 127 972 972 981 271 279 872 987 297 198
271 982 789 172 819 279 879 217

5) 930 249 923 904 923 940 929 909 930 990 994 919
974 929 932 984 939 983 967 947

3.1: Homework

1) How hot does it get in Death Valley? The following data are taken from a study conducted by the National Park System, of which Death Valley is a unit. The ground temperatures (°F) were taken from May to November in the vicinity of Furnace Creek.

146 152 168 174 180 178 179
180 178 178 168 165 152 144

Compute the mean, median, and mode for these ground temperatures.

2) How large is a wolf pack? The following information is from a random sample of winter wolf packs in regions of Alaska, Minnesota, Michigan, Wisconsin, Canada, and Finland (Source: *The Wolf*, by L. D. Mech, University of Minnesota Press). Winter pack size:

13 10 7 5 7 7 2 4 3
2 3 15 4 4 2 8 7 8

Compute the mean, median, and mode for the size of winter wolf packs.

3) The *Maui News* gave the following costs in dollars per day for a random sample of condominiums located throughout the island of Maui.

89	50	68	60	375	55	500	71	40	350
60	50	250	45	45	125	235	65	60	130

(a) Compute the mean, median, and mode for the data.

(b) Compute a 5% trimmed mean for the data, and compare it with the mean computed in part a. Does the trimmed mean more accurately reflect the general level of the daily rental costs?

(c) If you were a travel agent and a client asked about the daily cost of renting a condominium on Maui, what average would you use? Explain. Is there any other information about the costs that you think might be useful, such as the spread of the costs?

3.2: Measures of Variation

_____ – spread of the data.

Measures of Variance:

_____ – the difference between the largest and smallest values of a distribution.
**does not tell us how much other values vary from one another.

_____ – is a measurement that will give you a better idea of how the data entries differ from the mean.
****formula differs depending on whether you are using an entire population or just a sample.

- x is any entry in the distribution, x bar is the mean, and *n* is the number of entries.

*** Notice that the standard deviation uses the difference between each entry x and the mean x bar. The quantity (x – x bar) will be _____ if the mean is greater than the entry. If you take the sum Σ (x – x bar) then the negative values

will _____ the positive values, leaving you with a variation measure of 0 even if some entries vary greatly from the mean. Once the quantities become _____, the possibility of having some negative values in the sum is eliminated.

To Solve a Standard Deviation Problem:

1. Calculate *n*, the number of entries.
2. Calculate x bar, the mean, by using

3. Create a table using three columns, x, x – x bar, and $(x - \text{x bar})^2$.
4. Add all of the values in the $(x - \text{x bar})^2$ column.
5. To obtain the variance, _____ the sum from step 4 by $n - 1$.
6. Use your calculator to take the _____ of the variance.

A random sample of seven New York plays gave the following information about how long each play ran on Broadway (in days):

12 45 36 118 50 7 20

a. Find the range.
b. Find the sample mean.
c. Find the sample standard deviation.

Solution:

Part A is rather simple, we know our largest value is 118 and our smallest value is 7. If we substitute that in our range formula we arrive at:

Part B is just asking for the sample mean. We add up all of our entries and divide by the total number of entries. We then arrive at a sample mean of 41.14 days.

Part C is where it gets a little tricky. Let's create a chart that breaks down the standard deviation formula.

Length of Broadway Plays (in days):

x	x – x bar	$(x - \text{x bar})^2$
7	7 – 41.14 = -34.14	1165.54
12		
20		
36		
45		
50		
118		
$\Sigma x = 288$		$\Sigma(x - \text{x bar})^2 =$

After we have completed this chart, we need to take care of the denominator of our formula, by figure out what *n* is equal to.

***n* = _____ therefore *n* – 1 = _____**

We will now take our $\Sigma(x - \text{x bar})^2$ = _____ and divide that by ***n* – 1** = _____.

What is the result? _____

If we think about it, this answer only gives us a **sample variance.** What do you think we should do to the result above to come up with the sample standard deviation? Why?

s = _____

Petroleum pollution in oceans is known to increase the growth of a certain bacteria. Brian did a project for his ecology class for which he made a bacteria count (per 100 milliliters in nine random samples of sea water. His counts gave the following readings:

17	23	18	19	21
16	12	15	18	

a. Find the range.
b. Find the sample mean.
c. Find the sample standard deviation.

In the process of tuna fishing, porpoises are sometimes accidentally caught and killed. A U.S. oceanographic institute wants to study the number of porpoises killed. Records from eight commercial tuna fishing fleets gave the following information about the number of porpoises killed in a three-month period:

6	18	9	0
15	3	10	2

a. Find the range.
b. Find the sample mean.
c. Find the sample standard deviation.

Black Hole Pizza Parlor instructs its cooks to put a "handful" of cheese on each large pizza. Random samples of six such handfuls were weighed. The weights to the nearest ounce were:

3 2 3 4 3 5

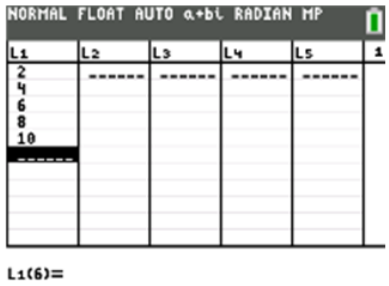
a. Find the mode, median, and mean weight of the handfuls of cheese.
b. Find the range and standard deviation of the weights.

c. A new cook used to play football and has large hands. His handful of cheese weighs 6 ounce. Replace the 2 ounce data value by 6 ounces. Recalculate the mode, median, and mean. Which average changed the most? Comment on the changes!

Calculator Instructions

1) STAT EDIT: Choice 1 on list of options

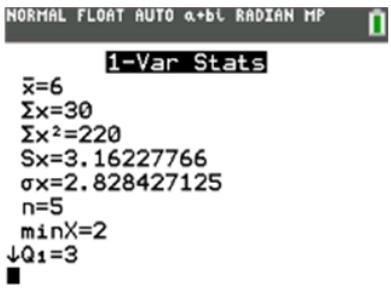
2) Enter the data in L1



3) STAT > CALC: Choice 1 (1-Var Stats)

4) σx - is the Population Standard Deviation

Sx - is Sample Standard Deviation



***Note that these are the same instructions as before, we are just looking for different information from the list.

Population Mean and Standard Deviation:

Until this point, we have mainly been working with random samples. However, we can work with the entire population, by computing the _____ (μ, Greek letter mu) and the _____ (σ, Greek letter sigma).

Formulas:

Population Mean:

Population Standard Deviation:

Where N is the number of data values in the population, x represents the individual data values of the population, μ is the same formula as x bar (sample mean), σ is the same as the formula for *s* (sample standard deviation).

To compute these two formulas by hand we will once again construct a computation table to guide us along the way. Our table will look like this:

x	$x - \mu$	$(x - \mu)^2$
$\Sigma x =$		$\Sigma(x - \mu)^2 =$

Bill has been training for the upcoming track season. He has been running the mile daily for the past week. His times were as follows (in minutes):

8.7 8.9 7.4 6 8.9 10 12.2

a. Calculate the population mean and population standard deviation.

The recent prices of SFP stock are indicated:

54.3 56.2 57.2 49.5 55.5 56.0 59.9

a) Calculate the range, mode, and median

1. Calculate the sample mean and sample standard deviation.
2. Calculate the population mean and population standard deviation.

On a recent exam, 10 students received the following scores:
86 77 99 100 86 86 82 95 99 100

1. Calculate the range, mode, and median.
2. Calculate the sample mean and standard deviation.
3. Calculate the population mean and population standard deviation.

Coefficient of Variation:

It is often difficult to use our standard deviation formula to compare measurements from different populations. Due to this fact, statisticians produced the _____.

The coefficient of variation expresses the standard deviation as _____ of what is being measured relative to the sample or population mean.

If x bar and s represent the sample mean and the sample standard deviation, then the coefficient of variation (CV) is defined to be:

If μ and σ represent the population mean and standard deviation, then the coefficient of variation CV is defined to be:

*** Notice that the numerator and denominator in the definition of CV have the same units, so CV itself has no units of measurement. This gives us the advantage of being able to directly compare the variability of 2 different populations using the coefficient of variation.

To Solve a CV Problem:

1. Calculate the _____.
2. Calculate the _____.
3. Use the formulas above to calculate the coefficient of variation (CV).

During April of 1999, the daily closing of the ABCD, WXY, and Z-corp, gave the following information:

	ABCD	WXYZ	Z-corp.
Mean values for April 1999	134.4	179.5	98.6
Standard deviation for July 1999	2.6	3.77	3.72

a. For each stock, compute the coefficient of variation.
b. Comment on the results of each stock.

Terrier and SFP are two stocks traded on the New York Stock Exchange. For the past few weeks you recorded the Friday closing price (dollars per share):

Terrier:	32	35	34	36	31	39
SFP:	51	55	56	52	55	52

a. Compute the mode, median, and mean for Terrier.
b. Compute the mode, median, and mean for SFP.
c. Compute the range, sample standard deviation, and sample variance for Terrier.
d. Compute the range, sample standard deviation, and sample variance for SFP.
e. Compute the coefficient of variation for both Terrier and SFP. Compare the results and explain the meaning of these numbers.

One of the responsibilities of John's job in the antique shop is to keep track of the closing price of a certain portrait. His recorded over the past ten weeks are as follows (in dollars):

89	94	99	95	96
95	88	96	96	96

a. Compute the mode, median, and mean.
b. Compute the range, sample standard deviation, and sample variance.
c. Compute the coefficient of variation.

The park ranger has been keeping track of the number of endangered species in the park each month. His ten month data is as follows:

56	55	53	51	50
49	47	45	45	44

a. Compute the mode, median, and mean.
b. Compute the range, sample standard deviation, and sample variance.
c. Compute the coefficient of variation.
d. What do you notice about the numbers?

Chebyshev's Theorem:

P.L. Chebyshev – Russian Mathematician who lives from 1821 – 1894. He was a professor at the University of St. Petersburg, where he did a great deal of important work in both pure and applied mathematics. The most surprising aspect of Chebyshev's theorem is that it applies to any and all distributions of data values.

_____: For any set of data (either population or sample) and for any constant *k* _____ than 1, the proportion of the data that must lie within *k* standard deviations on either side of the mean is at least

In ordinary words, Chebyshev's Theorem says the following about sample or population data:
1. Start at the _____.
2. Back off *k* standard deviations _____ the mean and then advance *k* standard deviations _____ the mean.
3. The fractional part of the data in the interval described will be at least $1 - 1/k^2$ (we assume k > 1).

Minimal Percentage of Data Falling within *k* Standard Deviations of the Mean:

k	*2*	*3*	*4*	*5*	*10*

*** Take k^2 and multiply it by the standard deviation. Add the result to and subtract the result from the mean to give you the interval.

Each year the National Weather Bureau produces information on the number of hurricanes in the U.S. The total number of hurricanes reported globally between the years of 1980 and 2006 are as follows:

75	79	83	86	71	44	86
77	87	100	94	66	40	72
61	42					

1. Calculate the sample mean and sample standard deviation.

2. Use Chebyshev's Theorem to find an interval centered about the mean in which you would expect 75% of the years to fall.

3. Use Chebyshev's Theorem to find an interval centered about the mean in which you would expect 88.9% of the years to fall.

4. Use Chebyshev's Theorem to find an interval centered about the mean in which you would expect 96% of the years to fall.

Based on the following data, answer the questions below:

89	47	90	82	37	48	92	37	40	72
34	57	43	89	75	30	98	24	75	80
97	58	90	75	98	04	75	89	03	72
58	90	74	07	54	38	97	58	93	47
09	57	48	75	39	82				

1. Calculate the sample mean and sample standard deviation.

2. Use Chebyshev's Theorem to find an interval centered about the mean in which you would expect 75% of the years to fall.

3. Use Chebyshev's Theorem to find an interval centered about the mean in which you would expect 88.9% of the years to fall.

4. Use Chebyshev's Theorem to find an interval centered about the mean in which you would expect 96% of the years to fall.

Over the last decade, Amazon.com has sold the following number of books (in millions):

103	106	114	177	111
162	148	119	120	144

1. Calculate the sample mean and sample standard deviation.

2. Use Chebyshev's Theorem to find an interval centered about the mean in which you would expect 75% of the years to fall.

3. Use Chebyshev's Theorem to find an interval centered about the mean in which you would expect 93.8% of the years to fall.

4. Use Chebyshev's Theorem to find an interval centered about the mean in which you would expect 99% of the years to fall.

3.2: Homework

1) In this problem, we explore the effect on the standard deviation of adding the same constant to each data value in a data set. Consider the data set 5, 9, 10, 11, 15.
(a) Use a table, or a calculator to compute *sx*.

(b) Add 5 to each data value to get the new data set 10, 14, 15, 16, 20. Compute *sx*.

(c) Compare the results of parts (a) and (b). In general, how do you think the standard deviation of a data set changes if the same constant is added to each data value?

2) Do bonds reduce the overall risk of an investment portfolio? Let *x* be a random variable representing annual percent return for Vanguard Total Stock Index (all stocks). Let *y* be a random variable representing annual return for Vanguard Balanced Index (60% stock and 40% bond). For the past several years, we have the following data.

x:	11	0	36	21	31	23	24	-11	-11	-21
y:	10	-2	29	14	22	18	14	-2	-3	-10

(a) Compute $\sum x$, and $\sum y$

(b) Use the results of part (a) to compute the sample mean, and standard deviation for *x* and for *y. (you may use a calculator)*

(c) Compute a 75% Chebyshev interval around the mean for *x* values and also for *y* values. Use the intervals to compare the two funds.

(d) Compute the coefficient of variation for each fund. Use the coefficients of variation to compare the two funds. If *s* represents risks and represents expected return, then can be thought of as a measure of risk per unit of expected return. In this case, why is a smaller *CV* better? Explain.

3) Kevlar epoxy is a material used on the NASA Space Shuttle. Strands of this epoxy were tested at the 90% breaking strength. The following data represent time to failure (in hours) for a random sample of 50 epoxies. Let *x* be a random variable representing time to failure (in hours) at 90% breaking strength.

0.54	1.80	1.52	2.05	1.03	1.18	0.80	1.33	1.29	1.11
3.34	1.54	0.08	0.12	0.60	0.72	0.92	1.05	1.43	3.03
1.81	2.17	0.63	0.56	0.03	0.09	0.18	0.34	1.51	1.45
1.52	0.19	1.55	0.02	0.07	0.65	0.40	0.24	1.51	1.45
1.60	1.80	4.69	0.08	7.89	1.58	1.64	0.03	0.23	0.72

(a) Find the range.

(b) Use a calculator to verify that $\sum x = 62.11$ and $\sum x^2 \approx 164.23$.

(c) Use the results of part (b) to compute the sample mean, and sample standard deviation for the time to failure. *(you may use a calculator)*

(d) Use the results of part (c) to compute the coefficient of variation. What does this number say about time to failure? Why does a small *CV* indicate more consistent data, whereas a larger *CV* indicates less consistent data? Explain.

4) Pax World Balanced is a highly respected, socially responsible mutual fund of stocks and bonds (see Viewpoint). Vanguard Balanced Index is another highly regarded fund that represents the entire U.S. stock and bond market (an index fund). The mean and standard deviation of annualized percent returns are shown below. The annualized mean and standard deviation are based on the years 1993 through 2002.

Pax World Balanced:	$\bar{x}$ = 9.58%;	s = 14.05%
Vanguard Balanced Index:	$\bar{x}$ = 9.02%;	s = 12.50%

(a) Compute the coefficient of variation for each fund. If represents return and s represents risk, then explain why the coefficient of variation can be taken to represent risk per unit of return. From this point of view, which fund appears to be better? Explain.

(b) Compute a 75% Chebyshev interval around the mean for each fund. Use the intervals to compare the two funds. As usual, past performance does not guarantee future performance.

3.3: Mean and Standard Deviation of Grouped Data

If you have many data values, it can be very time consuming to compute the mean and standard deviation. This includes when you are able to use the calculator, since you still have to put your data values into a list. In many cases a close approximation to the mean and standard deviation is all that is needed. It is not difficult to approximate these two values from a _____.

Procedure:

1) Make a frequency table corresponding to the histogram.
2) Compute the _____ for each class and call it *x*.
3) Count the number of _____ in each class and denote the number by *f*.
4) _____ the number of entries from each class together to find the total number of entries *n* in the sample distribution.

Sample Mean for a Frequency Distribution

,where x is the midpoint of a class, f is the number of entries in that class, n is the total number of entries in the distribution, and the summation Σ is over all classes in the distribution.

Sample Standard Deviation for a Frequency Distribution:

To break these formulas down, you should construct a table with the following columns:

x	f	xf	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
	$\Sigma =$	$\Sigma =$			$\Sigma =$

For each of the following, use the given table to approximate the sample mean and sample standard deviation:

Class	Frequency
0 – 50	254
51 – 100	361
	$\Sigma =$

Class	**Frequency**
0 – 3	10
4 – 7	25
8 – 11	36
12 – 15	45
16 – 19	18
20 - 23	29
	Σ =

Class	Frequency
0 – 5	404
6 – 11	365
12 – 17	158
18 – 23	299
24 – 29	365
30 – 35	225
36 - 41	179
	Σ =

Weighted Average:

There are instances where we would like to take an average of data, but assign more importance _____ to some of these numbers.

If we view the weight of a measurement as "frequency" then we discover that the formula for the mean of a frequency distribution gives us the weighted average.

where w is the weight of the data value x.

WEIGHTED AVERAGE

Suppose your midterm test score is 83 and your final exam score is 95. Using weights of 40% for the midterm and 60% for the final exam, compute the weighted average of your scores. If the minimum average for an A is 90, will you earn an A?

SOLUTION: By the formula, we multiply each score by its weight and add the results together. Then we divide by the sum of all the weights. Converting the percentages to decimal notation, we get

$$\text{Weighted average} = \frac{83(0.40) + 95(0.60)}{0.40 + 0.60}$$

$$= \frac{33.2 + 57}{1} = 90.2$$

Your average is high enough to earn an A.

Suppose you were being evaluated in a speech competition. The following criteria will be evaluated: punctuality, performance, delivery, length, and pronunciation. You are being evaluated on a scale of 1 – 10 with certain weights being assigned to each category as follows:

Category	Score	Weight
Punctuality	8	5%
Performance	7	30%
Delivery	3	30%
Length	4	10%
Pronunciation	6	25%

If the minimum score to advance to the next round is 5, will you advance?

Your grade in a certain class will be based on the following with the weights shown: tests (45%), quizzes (20%), homework (15%), attendance (15%), and class participation (5%). You receive the following grades in each category: tests – 80, quizzes – 95, homework – 90, attendance – 78, and class participation – 100. What is your grade?

On the first day of college your bio-molecular physics professor hands you a rubric on how you will be graded. You notice that attendance, projects, presentations, and a final exam will be evaluated. The weights assigned to each of these are: attendance (5%), tests (20%), projects (30%), presentations (30%), and final exam (15%). You have been given the following grades in each area: attendance – 100, tests – 87, projects – 95, presentations – 91, and final exam – 89. You are currently on scholarship and need to receive an A in every class. In this class an A can be obtained by getting a 91 or above. Do you maintain your scholarship for the following semester?

Two stocks are being evaluated by an investor. He will select the stock that has a higher average in all of the following categories: dividend (20%), security (50%), and growth (30%). He studies ESPN and FSNY and gives the following ratings on a scale of 1 – 20:

Category	ESPN	FSNY
Dividend	17	14
Security	6	12
Growth	11	13

Which stock the investor select and why?

3.3: Homework

1) In your biology class, your final grade is based on several things: a lab score, scores on two major tests, and your score on the final exam. There are 100 points available for each score. However, the lab score is worth 25% of your total grade, each major test is worth 22.5%, and the final exam is worth 30%. Compute the weighted average for the following scores: 92 on the lab, 81 on the first major test, 93 on the second major test, and 85 on the final exam.

2) At General Hospital, nurses are given performance evaluations to determine eligibility for merit pay raises. The supervisor rates the nurses on a scale of 1 to 10 (10 being the highest rating) for several activities: promptness, record keeping, appearance, and bedside manner with patients. Then an average is determined by giving a weight of 2 for promptness, 3 for record keeping, 1 for appearance, and 4 for bedside manner with patients. What is the average rating for a nurse with ratings of 9 for promptness, 7 for record keeping, 6 for appearance, and 10 for bedside manner?

3) What are the big corporations doing with their wealth? One way to answer this question is to examine profits as percentage of assets. A random sample of 50 *Fortune 500* companies gave the following information.

Profit as percentage of assets	8.6–12.5	12.6–16.5	16.6–20.5	20.6–24.5	24.6–28.5
Number of companies	15	20	5	7	3

Estimate the sample mean and sample standard deviation for profit as percentage of assets.

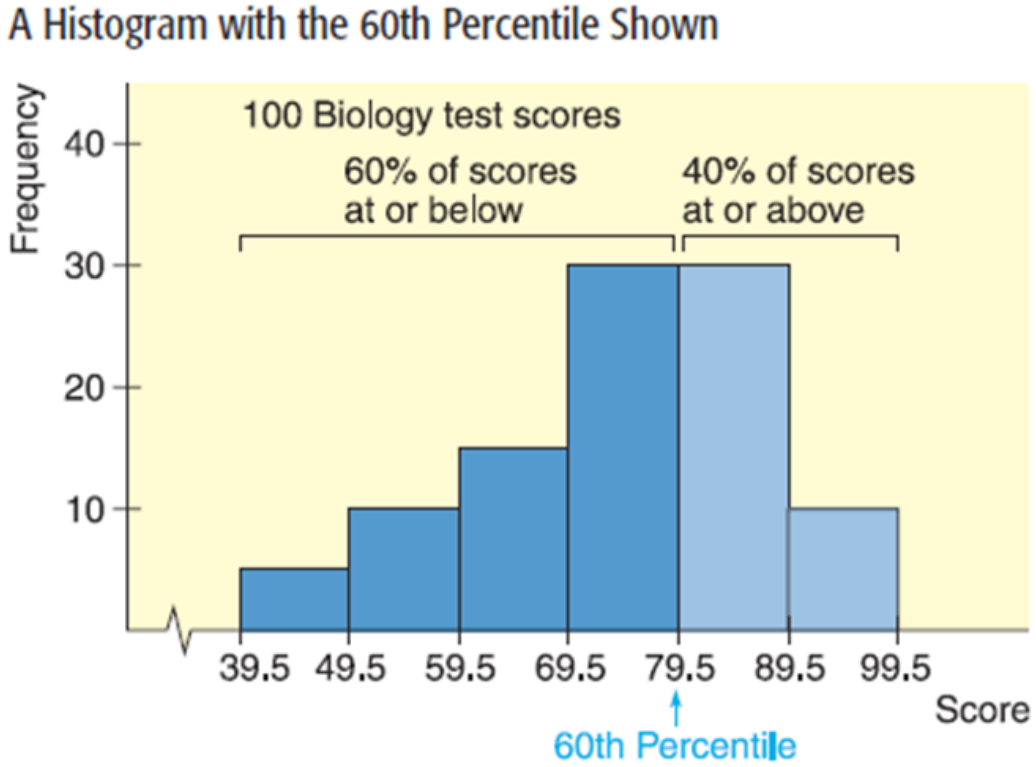
3.4: Percentiles and Box-and-Whisker Plots

Due to some cases where our data distributions are heavily skewed or even bimodal, we are usually better off using the relative position of the data as opposed to exact values.

We have studied how the median is an average computed using relative position of the data. If we say that the median is 27, then we know that half (50%) of the data falls above 27 and half (50%) of the data falls below 27. The median is an example of a percentile (50^{th} percentile).

Percentiles

For whole numbers P (where $1 \leq P \leq 99$) the Pth percentile of a distribution is a value such that P% of the data fall at or below it.



_____ are the summary measures that divide a ranked data set into 100 equal parts. Each (ranked) data set has 99 percentiles that divide it into 100 equal parts. The data set should be ranked in increasing order to compute percentiles. The kth percentile is denoted P_k, where k is an integer in the range 1 to 99. For instance, the 25^{th} percentile is denoted by P_{25}.

The kth percentile, P_k, can be defined as a value in a data set such that about k% of the measurements are smaller than the value of P_k and about (100 – k)% of the measurements are greater than the value of P_k.

Calculating Percentiles:

The approximate value of the kth percentile, denoted by P_k is:

P_k = Value of the (kn ÷ 100)th term in a ranked data set

where k denotes the number of the _____ and n represents the _____

Example:

Use the following data values:

284 586 987 412 256 541 312 251 444 695

Find the position of the

1) 42^{nd} percentile

2) 53^{rd} percentile

3)88^{th} percentile

Finding Percentile Rank of a Value:

We can also calculate the _____ for a particular value x_1 of a data set by using the formula given below. The percentile rank of x_1 gives the percentage of values in the data set that are less than x_1.

Example:

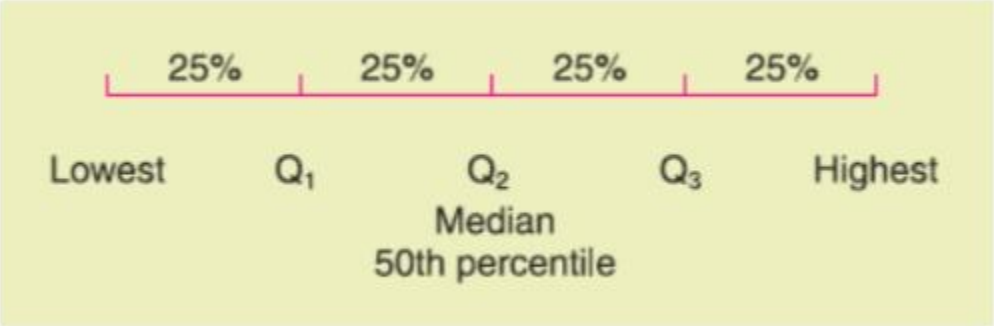
Use the data set from above to find the following

1) the percentile rank of 312

2) the percentile rank of 444

3) the percentile rank of 586

Quartiles – percentiles which divide the data into _____.
Example:
1st quartile = 25th percentile
2nd quartile = median
3rd quartile = 75th percentile



Interquartile Range:
A useful measure of data spread utilizing relative position is the interquartile range **(IQR)**. This is the difference between the 3rd and 1st quartiles.

This range tells us the spread of the _____ of the data.

The following data give the number of keyboards assembled at the Twentieth Century Electronics Company for a sample of 25 days.

45	52	48	41	56	46	44	42	48	53	51	53	51
48	46	43	52	50	54	47	44	47	50	49	52	

a) Calculate the values of the three quartiles and the interquartile range.

b) Determine the approximate value of the 53^{rd} percentile.

c) Find the percentile rank of 50.

Procedure to Compute Quartiles:

1. Rank the data from smallest to largest.
2. Find the _____ (2^{nd} quartile).
3. The first quartile (Q_1) is then the median of the _____ of the data; that is, it is the median of the data falling below Q_2 (and not including Q_2).
4. The third quartile Q_3 is the median of the _____ of the data; that is, it is the median of the data falling above Q_2 (and not including Q_2).

For each of the following data sets, calculate the median rank, median, 1st quartile, 3rd quartile, and interquartile range:

1) 100 97 106 87 94 102 101 99 86 78 96 56
80 106 111 87 88 80 96 98 96 91

2) 78 89 56 67 45 67 89 78 55 44 78 55
34 90 66 54 78 97 67 89 76 78 89 88

3) 67 215 56 81 96 200 197 196 133 145 99 100
154 167 166 189 177 189 199 222 221 67 71 98
87 78

4) 333 456 399 345 390 411 400 405 415 388 327
378 345 377 389 378 322 267 400 409 467 422

Box-and-Whisker Plots:

The quartiles, together with the low and high data values give us a very useful _____

Five Number Summary:

1) Lowest Value
2) Q_1
3) Median
4) Q_2
5) Highest Value

We use all five numbers to create a graphical sketch of the data called a _____**.** These plots are a useful way to describe data for exploratory data analysis (EDA).

To Construct a Box-and-Whisker Plot:

1) Draw a horizontal scale to include the highest and lowest data values.
2) To the right of the scale draw a box from Q_1 to Q_3.
3) Include a solid line through the box at the median level.
4) Draw solid lines, called whiskers, from Q_1 to the lowest value and from Q_3 to the highest value.

1) 45 67 34 78 29 68 32 64 78 96 54 05
54 97 65 94 86 09 05 46 79 05 69 80
76 09 76 98 07 69

2)

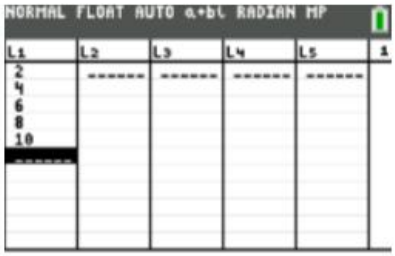
64	39	75	86	34	57	64	37	60	38	92	14
83	74	97	29	37	43	97	98	72	49	87	39
84	79	82	37	49	83	74	98	32	74	74	93

3)

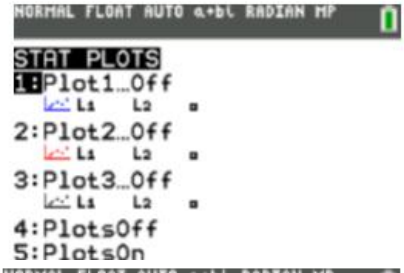
65	74	86	39	86	57	89	36	58	73	65	34
65	83	65	89	26	59	29	27	50	92	17	34
90	75	98	37								

Calculator Instructions

1) With data entered in to L1



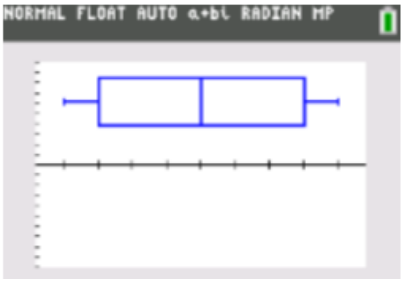
2) 2ND Y=: Choose option 1



3) Select ON, and Select the 5th option from the Type list

4) ZOOM: Choose option 9

5) Remember to turn off the Stat Plots and reset the zoom when finished. You can hit TRACE to find your 5 data points.



Try these examples using the Calculator:

1)

62	39	86	43	82	65	78	23	46
58	47	83	26	57	34	65	62	53
64	56	43	65	76	34	56	38	26
59	36	54	62	58	29	64	53	57
23	58	43	26	96	26	66	57	63
45	23	65						

2)

768	296	587	964	969	483	654	658	569
236	534	693	653	298	659	465	326	590
912	078	993	218	075	098	570	397	597
947	598	753	275	107	074	309	874	594
738	787	937	210	710	710	674	896	037
280	763	073	534	523	563	535	635	436
535	435	433	533	334	535	634	535	435
635	634	652						

3.4: Homework

1) The following data give the number of students suspended for bringing weapons to schools in the Tri-City School District for each of the past 12 weeks.

15	9	12	11	7	6
9	10	14	3	6	5

a) Calculate the values of the three quartiles and the interquartile range.

b) Determine the approximate value of the 55^{th} percentile.

c) Find the percentile rank of 7.

2) Another survey was done at Center Hospital to determine how long (in months) clerical staff had been in their current positions. The responses (in months) of 20 clerical staff members were

25	22	7	24	26	31	18	14	17	20
31	42	6	25	22	3	29	32	15	72

Make a box-and-whisker plot. Find the interquartile range.

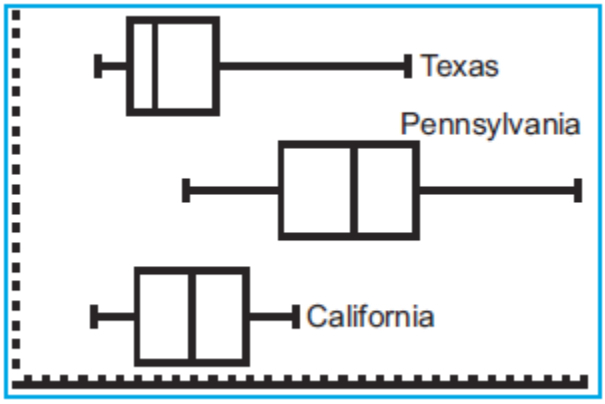
3) What percentage of the general U.S. population are high-school dropouts? The *Statistical Abstract of the United States*, 120th Edition, gives the percentage of high-school dropouts by state. For convenience, the data are sorted in increasing order.

5	6	7	7	7	7	8	8	8	8
8	9	9	9	9	9	9	9	10	10
10	10	10	10	10	10	11	11	11	11
11	11	11	11	12	12	12	12	13	13
13	13	13	13	14	14	14	14	14	15

(a) Make a box-and-whisker plot and find the interquartile range.

(b) Wyoming has a dropout rate of about 7%. Into what quartile does this rate fall?

4) *Consumer Reports* rated automobile insurance companies and gave annual premiums for top-rated companies in several states. The figure shows box plots for annual premiums for urban customers (married couple with one 17-year-old son) in three states. The box plots in the figure were all drawn using the same scale on a TI-84Plus/TI-83Plus calculator.



Five-Number Summaries for Insurance Premiums

(a)

```
1-Var Stats
↑n=10
 minX=2382
 Q1=2758
 Med=2991
 Q3=3652
 maxX=5715
```

(b)

```
1-Var Stats
↑n=10
 minX=3314
 Q1=4326
 Med=5116.5
 Q3=5801
 maxX=7527
```

(c)

```
1-Var Stats
↑n=10
 minX=2323
 Q1=2801
 Med=3377.5
 Q3=3966
 maxX=4482
```

a) Texas b) Pennsylvania c) California

(a) Which state has the lowest premium? Which state has the highest premium?

(b) Which state has the highest median premium?

(c) Which state has the smallest range of premiums? Which state has the smallest interquartile range?